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THESIS

HOW TO OPTIMIZE JOINT THEATER BALLISTIC MISSILE DEFENSE

by

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March 2004

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Many potential adversaries seek, or already have theater ballistic missiles capable of threatening targets of interest to the United States. The U.S. Missile Defense Agency and armed forces are developing and fielding missile interceptors carried by many different platforms, including ships, aircraft, and ground units. Given some exigent threat, the U.S. must decide where to position defensive platforms and how they should engage potential belligerent missile attacks. To plan such defenses, the Navy uses its Area Air Defense Commander (AADC) system afloat and ashore, the Air Force has its Theater Battle Management Core Systems (TBMCS) used in air operations centers, and the Missile Defense Agency uses the Commander's Analysis and Planning Simulation (CAPS). AADC uses a server farm to exhaustively enumerate potential enemy launch points, missiles, threatened targets, and interceptor platform positions. TBMCS automates a heuristic cookie-cutter overlay of potential launch fans by defensive interceptor envelopes. Given a complete missile attack plan and a responding defense, CAPS assesses the engagement geometry and resulting coverage against manually prepared attack scenarios and defense designs. We express the enemy courses of action as a mathematical optimization to maximize expected damage, and then show how to optimize our defensive interceptor pre-positioning to minimize the maximum achievable expected damage. We can evaluate exchanges where each of our defending platform locations and interceptor commitments are hidden from, or known in advance by the attacker. Using a laptop computer we can produce a provably optimal defensive plan in minutes.

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HOW TO OPTIMIZE JOINT THEATER BALLISTIC MISSILE DEFENSE

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ABSTRACT

Many potential adversaries seek, or already have theater ballistic missiles capable of threatening targets of interest to the United States. The U.S. Missile Defense Agency and armed forces are developing and fielding missile interceptors carried by many different platforms, including ships, aircraft, and ground units. Given some exigent threat, the U.S. must decide where to position defensive platforms and how they should engage potential belligerent missile attacks. To plan such defenses, the Navy uses its Area Air Defense Commander (AADC) system afloat and ashore, the Air Force has its Theater Battle Management Core Systems (TBMCS) used in air operations centers, and the Missile Defense Agency uses the Commander's Analysis and Planning Simulation (CAPS). AADC uses a server farm to exhaustively enumerate potential enemy launch points, missiles, threatened targets, and interceptor platform positions. TBMCS automates a heuristic cookie-cutter overlay of potential launch fans by defensive interceptor envelopes. Given a complete missile attack plan and a responding defense, CAPS assesses the engagement geometry and resulting coverage against manually prepared attack scenarios and defense designs. We express the enemy courses of action as a mathematical optimization to maximize expected damage, and then show how to optimize our defensive interceptor pre-positioning to minimize the maximum achievable expected damage. We can evaluate exchanges where each of our defending platform locations and interceptor commitments are hidden from, or known in advance by the attacker. Using a laptop computer we can produce a provably optimal defensive plan in minutes.

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LIST OF ACRONYMS

(This is the lexicon of theater ballistic missile defense. Not all of these terms appear herein, but they are essential to deciphering our references.)

AADC	Area Air Defense Commander
AADC-S	Area Air Defense Commander - System
AAW	Anti-Air Warfare
ABMA	Automated Battle Management Aid
ABL	Airborne Laser
ABT	Air-breather Threat
ACA	Airspace Control Authority
ACTD	Advanced Concept Technology Demonstration
ADAAM	Air-Directed Air-to-Air Missile
ADP	Air Defense Plan
ADSAM	Air-Directed Surface-to-Air Missile
AEW	Airborne Early Warning
APOD	Aerial Port of Debarkation
ASCM	Anti-Ship Cruise Missile
ASMD	Anti-Ship Missile Defense
ASW	Anti-Submarine Warfare
ATD	Advanced Technology Demonstration
ATR	Automated Target Recognition
BMC^4I	Battle Management / Command, Control, Communications, Computers & Intelligence
BMD	Ballistic Missile Defense
C2	Command and Control
C^4I	Command, Control, Communications, Computers & Intelligence
C4ISR	Command, Control, Communications, Computers, Intelligence, Surveillance and Reconnaissance
CA	Critical Asset
CBR	Chemical, Biological and Radiological
CCID	Composite Combat Identification
CEC	Cooperative Engagement Capability
CID	Combat Identification
CM	Cruise Missile
CMD	Cruise Missile Defense
CNA	Computer Network Attack
CNO	Chief of Naval Operations
COA	Course of Action
COP	Common Operational Picture

COTS	Commercial Off-the-Shelf
CRD	Capstone Requirements Document
CTE	Common Threat Evaluation
CTP	Common Tactical Picture
CVBG	Aircraft Carrier Battle Group
CVRT	Criticality, Vulnerability, Reconstitutability & Threat
DAL	Defended Asset List
DCA	Defensive Counter air
DEW	Directed Energy Weapon
DoD	Department of Defense
DOTMLP-F & Facilities	Doctrine, Organization, Training, Material, Leadership, Personnel
DRM	Design Reference Mission
DSC	Distributed Sensor Coordination
DWC	Distributed Weapons Coordination
EA	Electronic Attack
ECOA	Enemy Course of Action
EOB	Enemy Order of Battle
ES	Electronic Support
EW	Electronic Warfare
FDO	Flexible Deterrent Option
FEZ	Fighter Engagement Zone
FOB	Friendly Order of Battle
FNC	Future Naval Capability
GBI	Ground Based Interceptor
GBR	Ground Based Radar
HVA	High Value Asset
HVAA	High Value Airborne Asset
IBCT	Interim Brigade Combat Team
ID	Identification
IFF	Identification Friend or Foe
IFC	Integrated Fire Control
IO	Information Operations
IPB	Intelligence Preparation of the Battlefield (or Battlespace)
IRBM	Intermediate Range Ballistic Missile
IRST	Infrared Search and Track
IW	Information Warfare

JCTN	Joint Composite Tracking Network
JDN	Joint Data Network
JEZ	Joint Engagement Zone
JFACC	Joint Force Air Component Commander
JFC	Joint Force Commander
JFMCC	Joint Force Maritime Component Commander
JIAD	Joint Integrated Air Defense
JPN	Joint Planning Network
JTAMD	Joint Theater Air and Missile Defense
JTAMDO	Joint Theater Air and Missile Defense Organization
LACM	Land Attack Cruise Missile
LACMD	Land Attack Cruise Missile Defense
MD	Missile Defense
MEZ	Missile Engagement Zone
MIDAS	Multifunction Infrared Distributed Aperture System
MOE	Measure(s) of Effectiveness
MOP	Measure(s) of Performance
MOS	Measure(s) of Success
MOU	Memorandum of Understanding
MRBM	Medium Range Ballistic Missile
NCO	Network Centric Operations
NTW	Navy Theater-Wide (Ballistic Missile Defense Program)
OCA	Offensive Counter Air
OCM	Overland Cruise Missile
OCMD	Overland Cruise Missile Defense
OMFTS	Operational Maneuver From The Sea
OPDEC	Operational Deception
OPSEC	Operational Security
P_I	Probability of Interception
P_{ID}	Probability of Identification
P_{KSS}	Probability of Single-Shot Kill
P_{NEG}	Probability of Negation
PAC-3	PATRIOT Advanced Capability 3
POD	Port of Debarkation
PSR	Preferred Shooter Recommendation
RADC	Regional Air Defense Commander
RMP	Radar Modernization Program
ROE	Rules of Engagement
RSTA	Reconnaissance, Surveillance, Targeting and Assessment

S&T	Science and Technology
SADC	Sector Air Defense Commander
SHORAD	Short Range Air Defense
SIAP	Single Integrated Air Picture
SM	STANDARD Missile
SOF	Special Operations Forces
SPAWARSYSCOM	Space and Naval Warfare Systems Command
SPOD	Seaport of Debarkation
SRBM	Short Range Ballistic Missile
STOM	Ship to Objective Maneuver
TAD	Theater Air Defense
TAMD	Theater Air and Missile Defense
TBM	Theater Ballistic Missile
TBMD	Theater Ballistic Missile Defense
TDMP	Test and Demonstration Master Plan
TEL	Transporter, Erector, Launcher
TEWA	Threat Evaluation / Weapon Assignment Project
THAAD	Theater High Altitude Air Defense
TLAM	Tomahawk Land Attack Missile
TM	Theater Missile
TMD	Theater Missile Defense
TST	Time Sensitive Target(ing)
UAV	Unmanned Aerial Vehicle
UCAV	Unmanned Combat Aerial Vehicle
UESA	UHF Electronically Scanned Array
UHF	Ultra High Frequency
WMD	Weapon(s) of Mass Destruction

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EXECUTIVE SUMMARY

Theater ballistic missiles (TBM) are capable of delivering high-explosive warheads as well as nuclear, biological, and chemical weapons, termed "weapons of mass destruction." Adversaries already have, or are developing these deadly devices, so the United States is developing defensive interceptors and tactics to maximize the probability of defeating all incoming ballistic missile threats. The goal is to thwart even a small nuclear, chemical or biological strike on a target city or military site.

Theater ballistic missile defense has become an important component of the Department of Defense research and development budget, whose requests for fiscal year 2005 exceed ten billion dollars for joint missile defense programs.

Soon a joint forces commander will have at his disposal a number of defensive interceptors, including ground-based THAAD and PATRIOT missile batteries, sea-based AEGIS ships, and aircraft that will be used to intercept an anticipated enemy ballistic missile attack. The commander needs quick and accurate recommendations for the advantageous positioning of joint forces to intercept ballistic missile threats.

The following programmed systems will be used for ballistic missile defense. The Army's PATRIOT consists of a mobile launcher, a phased array air search and tracking radar, and various command and support vehicles and is capable of firing three types of interceptor missiles, providing terminal defense against ballistic missiles, cruise missiles and aircraft. The Army's Theater High Altitude Air Defense (THAAD) system will provide a midcourse-high altitude defense of ballistic missiles using a kinetic-kill interceptor. The Navy has deployed the Ticonderoga-class guided missile cruisers and Arleigh Burke-class guided missile destroyers. Each of these ships has the AEGIS SPY-1 phased array radar and can function as a ballistic missile interceptor platform. The Air Force is developing the Airborne Laser (ABL), a chemical laser housed in a 747 aircraft that will provide boost phase defense against ballistic missiles. The ABL is not considered for our planning scenario, however adding the ABL, or any other ballistic missile defense system, is trivial.

The Department of Defense is currently using these planning tools to mitigate the ballistic missile threat: the Area Air Defense Commander (AADC) System (AN/UYQ-89), the Theater Battle Management Core Systems (TBMCS) and the Commander's Analysis and Planning Simulation (CAPS).

The Navy's Area Air Defense Commander (AADC) System deployed on command ships USS BLUERIDGE, USS MOUNT WHITNEY, the AEGIS cruiser USS SHILOH and at the Joint National Integration Center (JNIC) in Colorado.

AADC consists of a planning and operations module that allows air defense commanders to plan and war-game many "what-if" scenarios, analyze proposed defensive interceptor positioning, and monitor current events in near real-time on a three-dimensional projection of the battle space. AADC uses a server farm to exhaustively enumerate defensive solutions that consist of every feasible attack combination of enemy launch point, defended asset and friendly interceptor platform position to a high degree of fidelity.

Theater Battle Management Core Systems (TBMCS) is used by U.S. Air Force air operations centers for theater-level planning in support of the Area Air Defense Commander. TBMCS supports strategic planning, air battle planning, mission preparation, mission execution and reporting and analysis on near real-time situations as they unfold. TBMCS automates a heuristic cookie-cutter overlay of potential launch fans by defensive interceptor envelopes; this heuristic suggests a face-valid solution, but one of unknown quality.

Commander's Analysis and Planning Simulation (CAPS) was developed by SPARTA, Inc. for the Missile Defense Agency (MDA) in 1993. CAPS is used to assess defense system capabilities and positioning, develop a defense design, and to test the performance of the defense design over a hypothesized, manually-prepared threat scenario with respect to a manually-prepared defense design.

All three of these *fielded* systems solve the complex problem of ballistic missile defense in very different ways, with varying degrees of fidelity, and with differing objectives.

We express enemy courses of action as a mathematical optimization to maximize expected damage, and then show how to optimize our defensive asset prepositioning to minimize the maximum achievable expected damage. The problem is to optimize defensive positioning for attack interception while (perhaps) assuming the attacker will observe these preparations and optimize attacks to exploit any weaknesses in these defenses. *Our objective is to minimize the maximum total expected damage to targets.*

The resulting mathematical formulation is an integer linear program that recommends optimal stationing locations and interdictions for defender assets by minimizing the enemies' ability to inflict damage. Defender optimal interdiction strategy accounts for the launch sites of the attack, the missile types used and the targets attacked. Additionally, we balance interceptor capabilities and defender platform inventory to minimize the expected damage inflicted by enemy ballistic missiles that penetrate the air defenses. Defender interdiction strategy is further constrained by linking interceptor capabilities to the oblate spherical triangle formed by the geographic coordinates of the attacker launch site, target, and defender location, which depend on the attack the enemy chooses. The result is an integer linear program that recommends optimal stationing positions for defender platforms that minimize the maximum expected damage of an enemy attack.

For planning purposes, we develop a North Korean scenario set in 2010 consisting of sixteen launch sites, five missile types threatening twelve targets in South Korea, Japan and Okinawa. Facing this threat we have two AEGIS cruisers, one AEGIS destroyer, one PATRIOT battery and one THAAD battery. Each defender platform is allocated a loadout combination of six interceptor types.

The results of our model include a maximal attack with no defense, a defense of this attack, a two-sided attack where we assume that the attacker and defender can see each other's preparations and react accordingly, and a two-sided attack where we are able to keep some of our defenders hidden from the attacker.

We propose a decision support tool that can offer provably optimal interception plans on a laptop computer in minutes. These integer linear programs can be solved

faster, and can be expected to find a near-optimal solution. The space and power requirements for our solution are trivial. The Joint Task Force's Area Air Defense Commander (AADC) and Regional Air Defense Commander (RADC) may use the decision support tool for initial defense planning and assessment. In addition it could provide insight to the ballistic missile defense (BMD) program officers in Washington.

I. THEATER MISSILE DEFENSE PARDIGM

A. THE THEATER BALLISTIC MISSILE THREAT

Theater ballistic missiles (TBM) are capable of delivering high-explosive warheads as well as nuclear, biological, and chemical weapons, termed "weapons of mass destruction." Adversaries already have, or are developing these deadly devices, so the United States is developing defensive interceptors and tactics to maximize the probability of defeating all incoming ballistic missiles. The goal is to thwart even a small nuclear, chemical or biological strike on a target city or military site.

Theater ballistic missile defense has become an important component of the Department of Defense research and development budget, whose requests for fiscal year 2005 exceed ten billion dollars for joint missile defense programs [DoD 2005].

Soon joint forces commanders will have a number of defensive interceptors, including ground-based THAAD and PATRIOT missile batteries, sea-based AEGIS ships, and aircraft, which will be used to intercept an anticipated enemy ballistic missile attack. The joint forces commander needs quick and accurate recommendations for the advantageous positioning of joint forces to intercept ballistic missile threats.

Figure 1. Current ballistic missile threats
Shown left to right, a few of the threat missiles in existence today: Scud-B Transporter-Erector-Launcher (TEL), a TEL firing a missile, and an Iranian fixed ballistic missile launcher.

B. A CASE STUDY: NORTH KOREA TODAY

North Korea is known to be developing and exporting ballistic missiles and missile technology and has numerous indigenous missile production facilities and prepared launching sites. North Korean weapons experts are developing longer-range intercontinental ballistic missiles (e.g. the Taep'o-Dong II) that, in the near future, will be capable of delivering chemical and biological agents as well as conventional and fission warheads to the western coast of the United States and Alaska [CIA 2001]. North Korea has announced that it has developed nuclear weapons. It is vital that we understand what effect a ballistic missile first strike could have on the theater in a potential conflict.

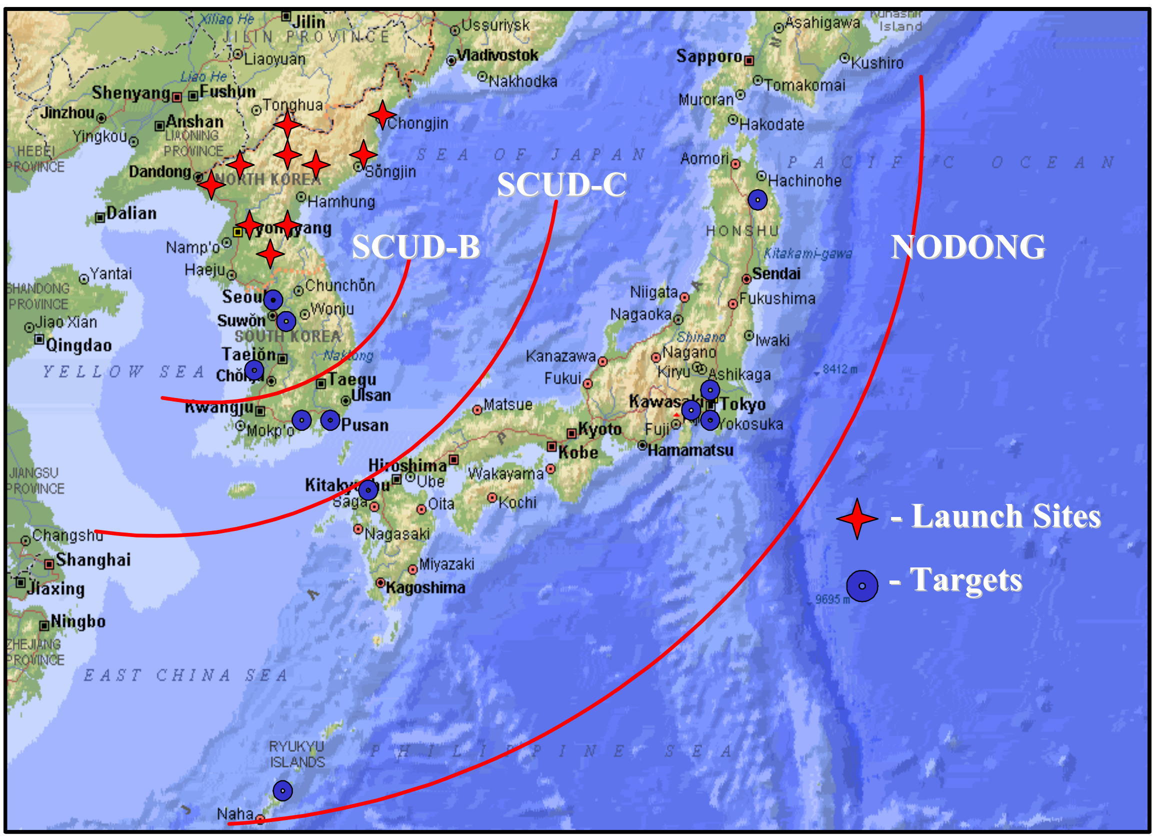

Figure 2. Depiction of maximum ranges of North Korean Scud-B, Scud-C, and No-Dong theater ballistic missiles.

Notice that all of Japan and Okinawa are vulnerable to the North Korean No-Dong missile.

C. CURRENT PROGRAMMED DEFENSE SYSTEMS

The following platforms are part of the joint missile defense architecture and are used in our scenario to represent the defending forces. Adding additional future systems, such as the Airborne Laser (ABL), is trivial.

Figure 3. Current programmed defense platforms

Shown left to right are a THAAD launch vehicle, an AEGIS Guided Missile Cruiser firing a standard missile, and a PATRIOT launch vehicle.

1. Army PATRIOT

The Army's PATRIOT is currently deployed and has seen use most recently in Operation Iraqi Freedom. PATRIOT provides a terminal defense against ballistic missiles, cruise missiles and aircraft. PATRIOT consists of a mobile launcher, a phased array air search and tracking radar, various command and support vehicles, and is capable of firing three interceptor missiles; the PAC-2, PAC-2 GEM, and PAC-3 [Jane's 2003c].

2. Army THAAD

The Army's THAAD (Theater High Altitude Air Defense) system is in development. THAAD will provide a midcourse-high altitude defense of ballistic missiles using a kinetic-kill interceptor. THAAD consists of a mobile launcher, a phased array air search and tracking radar, and command and support vehicles [Jane's 2003c].

3. Navy AEGIS

Navy AEGIS refers to deployed Ticonderoga-class guided missile cruisers and Arleigh Burke-class guided missile destroyers. Each of these ships has the AEGIS SPY-1 phased array radar and can function as a ballistic missile interceptor platform. Each ship class is currently deployed with Standard Missile-2 (SM2) variants that provide terminal defense against cruise missiles and aircraft. The Navy is developing the Standard Missile-3 (SM3), a kinetic-kill exo-atmospheric interceptor that will provide a midcourse defense from ballistic missiles [Jane's 2003b].

D. CURRENT PLANNING TOOLS

1. Area Air Defense Commander (AADC) System (AN/UYQ-89)

AADC has been developed for the Navy and is currently deployed on command ships USS BLUERIDGE, USS MOUNT WHITNEY, the AEGIS cruiser USS SHILOH and at the Joint National Integration Center (JNIC) in Colorado [Jane's 2003a].

AADC consists of a planning and operations module that allows air defense commanders to plan and war-game many "what-if" scenarios, to analyze proposed defensive interceptor positioning, and monitor current events in near real-time on a three-dimensional projection of the battle space.

AADC uses a server farm to exhaustively enumerate theater ballistic defensive solutions that consist of every feasible attack combination of enemy launch point, defended asset and friendly interceptor platform position to a high degree of fidelity. AADC positions forces to defend each asset at or above a specified probability of intercept before defending any lower-priority asset. AADC provides an estimate of defense coverage and an expected number of enemy missiles that will get through the defense design.

2. Theater Battle Management Core Systems (TBMCS)

TBMCS is used by U.S. Air Force air operations centers for theater-level planning in support of the Area Air Defense Commander.

TBMCS supports strategic planning, air battle planning, mission preparation, mission execution and reporting and analysis on near real-time situations as they unfold.

TBMCS automates a heuristic cookie-cutter overlay of potential launch fans by defensive interceptor envelopes; this heuristic suggests a face-valid solution, but one of unknown quality. There is no formal evaluation or optimization of a suggested defensive solution. Nor is there a mechanism to represent what the enemy would do in response to observing defensive preparations.

3. Commander's Analysis and Planning Simulation (CAPS)

CAPS was developed by SPARTA, Inc. for the Missile Defense Agency (MDA) in 1993. CAPS is currently hosted by theater ballistic missile planning cells of Central Command, European Command, Pacific Command, Strategic Command, the Naval Postgraduate School, and others, totaling over 50 sites.

CAPS is used to assess defense system capabilities and positioning, develop a defense design, and to test the performance of the defense design over a hypothesized, manually-prepared threat scenario with respect to a manually-prepared defense design [Sparta 2004]. The CAPS operator selects the best-looking defense design that protects defended targets with high probability and appears to maximize the number of potential engagements the defender has against the specified attack scenario.

All three of these *fielded* systems solve the complex problem of ballistic missile defense in very different ways, with varying degrees of fidelity, and with differing objectives.

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II. DEFENSIVE PLANNING WITH A NEW, TWO-SIDED OPTIMIZATION

We express the enemy courses of action as a mathematical optimization to maximize expected damage, and then show how to optimize our defensive asset prepositioning to minimize the maximum achievable expected damage.

A. TBMD TERMINOLOGY

1. Geography

Each *launch site* for attacking missiles is located by latitude and longitude. There may be an arbitrary number of dispersed launch sites. Inclusion of additional launch sites is trivial.

Each *target* vulnerable to enemy attack is located by latitude and longitude. There may be an arbitrary number of dispersed targets.

Each *target* has a *target value* that is agreed upon by the attacker and defender.

Each candidate *defender position* is located by latitude and longitude. Candidate positions are discretized into a field of grid points with desired geographic fidelity. E.g., a 0.1-degree mesh grid conveys fidelity no worse than 6 nautical miles between candidate positions. For planning purposes, a 1-degree (60 nm) grid is likely adequate.

2. Defended Asset List

Each defended asset *target* may be a point target, or an area target: this distinction influences how we account for expected attack damage.

3. Enemy Course of Action

Each enemy *missile* has a minimum and maximum range, and can hit any *target* within this range interval with a *probability of kill*. This probability can depend on the *missile* type, *target*, and *range* from *launch site* to *target*.

Each *launch site* is endowed with a given number of each enemy *missile* type.

An enemy *attack* consists of a maximal launch of enemy *missile(s)* from each enemy *launch site* against any or all vulnerable *targets*. The enemy goal is to maximize total expected target damage. If a vulnerable *target* is an area target, expected damage is additive by attack.

4. Friendly Course of Action

Each defender *class* consists of a given number of individual *platforms*, each endowed with a loadout of a given number of each type of *interceptor* weapon (e.g. missile). Each defending *platform* may be located at any candidate *defender position* secure and compatible for its *class*. (I.e., ships may only be positioned at sea, land units on compatible terrain, and air defenders in safe airspace.)

Any attacking *missile* may be engaged by any defending *platform* with an *intercept salvo* of any number of any types of *interceptor* missiles available on that defending *platform*. For planning purposes, and as a matter of effective tactical doctrine, we assume that the planned intercept of each enemy *missile* will be executed by just one defending *platform*. {In actual execution, however, this would not preclude defending platforms from providing a layered defense of defended targets.}

The probability that an *intercept salvo* will kill the attacking *missile* is a function of the attack launch location, missile type, target location, defender location, defending salvo, and the joint (*synergistic*) effectiveness of all intercepting missiles in that *intercept salvo*.

The geography of such an engagement can be depicted as an oblate spherical triangle, with apexes at the locations of *launch site*, *target*, and *defender position*. The probability of intercept salvo kill is then an arbitrary function of these geographic proximities and locations, the vulnerability of the attacking *missile* as it travels over its *flyout* trajectory, and the joint effectiveness of the entire *intercept salvo*. In practice, we might use either a mathematical approximation, and/or some engineering estimate tabulated in a "cross-range, down-range" probability of kill table for each intercept salvo type and missile altitude. *An intercept salvo kill probability does not rely on an independence assumption among and between individual intercepting missiles in the salvo.*

B. MATHMATICAL DEVELOPMENT OF ATTACKER AND DEFENDER OPTIMIZATIONS

There is a set of launch locations *L* and missile types *M*, with $missiles_{l,m}$ of type $m \in M$ at location $l \in L$ available, a set of targets *T*, with each $t \in T$ having target value $value_t$. An attack $a \in A$ consists of a launch from location $l_a \in L$ of a missile type $m_a \in M$ at a target $t_a \in T$ that will hit the target with probability k_a. The attacker's problem is to decide which missiles to launch at which targets to maximize total expected target damage.

Figure 4. Illustration of attack lexicon

An attack *a* consists of a location l_a launching a missile type m_a at a target t_a and expecting to hit the target with probability k_a. If the target has $value_t$, the expected damage of an attack is $k_a value_t$.

The defender has a set of defending platform classes, *C*, a set of defending platforms *P*, each of class $c_p \in C$, which can be pre-positioned at geographic locations *G*. Each platform class *c* has a set of locations $G_c \subseteq G$ at which it can be placed. Each class *c* carries defensive interceptor types *I*, with $loadout_{c,i}$, interceptors of type $i \in I$ available to platform class $c \in C$. An attack *a* can be engaged with alternate defensive actions *D*, where defense $d \in D$ launches $salvo_{a,c,d,i}$ interceptors and succeeds in thwarting the attack with probability $Pk_{a,c,g,d}$.

Figure 5. Illustration of defense lexicon

An attack *a* may be met with a defensive action launched by a class *c* platform in position *g* exercising defense alternative *d* that launches an interceptor type *i* (or, more generally, $salvo_{a,c,d,i}$ interceptors), and thwart the attack with probability $Pk_{a,c,g,d}$.

The defender's problem is to optimize defensive pre-positioning for attack interception while (perhaps) assuming the attacker will observe these preparations and optimize attacks to exploit any weaknesses in these defenses. *Our objective is to minimize the maximum total expected damage to targets.*

1. Indices and Index Sets

$l \in L$	launch location
$m \in M$	attacking missile type
$t \in T$	target, defended asset
$a \in A$	attack launching a missile at a target
l_a	launch location of attack a, $l_a \in L$
m_a	missile type launched in attack a, $m_a \in M$
t_a	target of attack a, $t_a \in T$
$p \in P$	defending platform
$c \in C$	defending platform class
c_p	class of platform p, $c_p \in C$
$g \in G$	candidate stationing location for a defending platform
$g_c \in G_c \subseteq G$	candidate stationing location for a defending platform of class c
$i \in I$	defensive interceptor type
$d \in D$	defense option
$b \in B$	Bender's iteration

2. Data [units]

$missiles_{l,m}$	launch location l supply of missile type m [missiles]
$value_t$	value of target t [value]
k_a	probability that attack a hits its target t_a [fraction]
$max\_missiles_t$	maximum number of missiles that can attack target t [missiles]
$loadout_{c,i}$	type i interceptors carried by a class c platform [interceptors]
$salvo_{a,c,d,i}$	defending against attack a, a class c platform exercising defense option d will use this number of type i interceptors [interceptors]
$Pk_{a,c,g,d}$	probability that attack a would be intercepted if a class c platform in location g exercises defense option d [fraction]

3. Variables [units]

Y_a 1 if attack conducted, 0 otherwise [binary]

$X_{p,g}$ 1 if platform *p* located at *g*, 0 otherwise [binary]

$R_{a,p,g,d}$ 1 if attack *a* is engaged by platform *p* in location *g* exercising defense option *d*, 0 otherwise [binary]

4. Minimax Optimization of Expected Damage [dual variables]

$$
\min_{\{X,R\}\in \mathrm{X}} \left\{
\begin{array}{llll}
\max\limits_{Y} & \sum\limits_{a} value_{t_a} \left(k_a \left[1 - \sum\limits_{p\in P, g\in G, d\in D} Pk_{a,c_p,g,d} R_{a,p,g,d} \right] \right) Y_a & & \text{(a0)} \\
s.t. & \sum\limits_{a|l=l_a \wedge m=m_a} Y_a \le missiles_{l,m} \qquad \forall l\in L, m\in M & \text{(a1)} & [\alpha_{l,m}] \\
 & \sum\limits_{a|t=t_a} Y_a \le max\_missiles_t \qquad \forall t\in T & \text{(a2)} & [\beta_t] \\
 & 0 \le Y_a \le 1 \qquad \forall a\in A & \text{(a3)} & [\gamma_a]
\end{array}
\right\}
$$

The attacker's objective is to maximize expected target damage (a0). Constraints (a1) limit the number of missiles available by launch location. Constraints (a2) limit the maximum number of missiles that can be launched at each target. Constraints (a3) limit each attack to at most one missile.

The objective expresses expected incremental target value inflicted as a consequence of each attacking missile. For a point target that might be substantially damaged or destroyed by any single attacking missile, there is no joint probability expression for surviving more than one, and this will multiply-credit target value unless we use constraint (a2) to allow at most one attacking missile. For an area target, such as a city or airfield, this is not an issue: each attacking missile damages its own, incremental expected target value.

5. Limits on defender's actions

The defender's actions are limited by $\{X,R\}\in \mathrm{X}$:

$$\sum_{g\in G} X_{p,g} \le 1 \qquad \forall p\in P \qquad \text{(d1)}$$

$$\sum_{p\in P} X_{p,g} \le 1 \qquad \forall g\in G \qquad \text{(d2)}$$

$$\sum_{p\in P, g\in G, d\in D} R_{a,p,g,d} \le 1 \qquad \forall a\in A \qquad \text{(d3)}$$

$$\sum_{a\in A, d\in D} salvo_{a,c_p,d,i} R_{a,p,g,d} \le loadout_{c_p,i} X_{p,g} \qquad \forall p\in P, g\in G_{c_p}, i\in I \qquad \text{(d4)}$$

$$X_{p,g}\in\{0,1\} \qquad \forall p\in P, g\in G \qquad \text{(d5)}$$

$$R_{a,p,g,d}\in\{0,1\} \qquad \forall a\in A, p\in P, g\in G, d\in D \qquad \text{(d6)}$$

Constraints (d1) limit each platform to occupy at most one grid location, constraints (d2) (optionally) limit each grid location to accommodate at most one platform, constraints (d3) allow at most one interception of each attack, constraints (d4) limit the number of engagements from each platform, from each grid point, and constraints (d5) and (d6) require binary decisions.

The attacker plans to maximize expected damage, and the defender plans to minimize the attacker's maximal expected damage.

If the defender can completely conceal his preparations, and the attacker acts optimally in ignorance of the defender's disposition of forces, we can emulate the results by solving the inner, optimal attack problem, and then the outer, optimal defense problem. This is the best the defender can hope to achieve if the attacker acts optimally.

Suppose the attacker can see the defender's preparations.

If the defender cannot see what the attacker does, he can still preposition his defending interceptors and commit their missiles to potential attacks as follows.

$$\max_{\{X,R\}\in \mathrm{X}} \sum_a k_a value_{t_a} (1-\sum_{p\in P, g\in G, d\in D} Pk_{a,c_p,g,d} R_{a,p,g,d}) \qquad (g0).$$

The defender positions platforms and commits interceptors to maximally protect target value.

Suppose both parties can observe each other's preparations. For fixed R, and if $missiles_{l,m}$ and max_salvo_t are integer, the attacker's maximizing problem is a linear program that will render an *intrinsically integer* optimal attack solution Y^*. Exploiting this key observation, we substitute the dual of the attacker's maximizing linear program yielding a mixed integer linear program.

6. Two-Sided Integer Linear Program to Minimize Maximum Achievable Expected Damage

$$\min_{\substack{\alpha,\beta,\gamma \\ X,R}} \sum_{l\in L,m\in M} missiles_{l,m}\alpha_{l,m} + \sum_{t\in T} max\_missiles_t \beta_t + \sum_{a\in A}\gamma_a \qquad (t0)$$

$$\text{s.t.} \quad \alpha_{l_a,m_a} + \beta_{t_a} + \gamma_a + \sum_{p\in P,g\in G,d\in D} k_a value_{t_a} Pk_{a,c_p,g,d} R_{a,p,g,d} \ge k_a value_{t_a} \qquad \forall a\in A \qquad (t1)$$

$$\sum_{g\in G} X_{p,g} \le 1 \qquad \forall p\in P \qquad (t2)$$

$$\sum_{p\in P} X_{p,g} \le 1 \qquad \forall g\in G \qquad (t3)$$

$$\sum_{p\in P,g\in G,d\in D} R_{a,p,g,d} \le 1 \qquad \forall a\in A \qquad (t4)$$

$$\sum_{a\in A,d\in D} salvo_{a,c_p,d,i} R_{a,p,g,d} - loadout_{c_p,i} X_{p,g} \le 0 \qquad \forall p\in P, g\in G_{c_p}, i\in I \qquad (t5)$$

$$\alpha_{l,m} \ge 0 \qquad \forall l\in L, m\in M \qquad (t6)$$

$$\beta_t \ge 0 \qquad \forall t\in T \qquad (t7)$$

$$\gamma_a \ge 0 \qquad \forall a\in A \qquad (t8)$$

$$X_{p,g} \in \{0,1\} \qquad \forall p\in P, g\in G_{c_p} \qquad (t9)$$

$$R_{a,p,g,d} \in \{0,1\} \qquad \forall a\in A, p\in P, g\in G, d\in D \qquad (t10)$$

Using a feasible binary defense location plan X^* and interception plan R^* from this integer linear program, we can recover the associated binary attack plan Y^* by solving the attacker's original maximizing linear program for this fixed X^* and R^*.

The integer linear program can be embellished as long as the modifications can be expressed linearly in $\{X,R\}\in \mathrm{X}$.

The distinguishing feature here is that the attacker can see the defender's preparations, and vice versa. Thus, the interpretation of the interception variables R^* changes from "shoot these missiles" to "commit these missiles to intercept that potential launch." Not all interceptions will actually be executed because the attacker may abandon attack options that he sees can and will be intercepted.

7. Relationship Between Two-sided Model and Game Theory

The similarity between our transparent two-sided (attacker-defender) model and a classical two-person zero sum (TPZS) game is compelling. We clearly model two opponents. Given fixed attack, defense, and engagement decisions, the resulting expected damage is a function of these quantities, and is therefore uniquely defined regardless of the order in which we fix them. Anything the attacker gains, the defender counts as a loss, so we have a zero-sum objective function. Finally, we have finite (albeit enormous) decision spaces for both attacker and defender. The transparent model can, in principle, be put into the context of TPZS game theory.

The attacker has as his decision space the set of all possible attacks he can mount (which correspond to all extreme points of the polyhedron defined by the constraints in the attacker subproblem), and the defender has all platform prepositions and engagements available to him. The cardinalities of both of these decision spaces are exponential in the basic parameters of the scenario, (number of launch sites, missiles, targets, platforms, grid locations, etc.), and therefore enumeration of the payoff matrix is practically impossible. Our transparent two-sided model enables us to determine a near-optimal solution to the min-max problem associated with this payoff matrix in a reasonable amount of time.

8. Bender's Decomposition of the Two-Sided Integer Linear Program

Applying Bender's decomposition to this two-sided integer linear program yields for any admissible candidate engagement plan $\overline{R}^{b}_{a,p,g,d}$ the linear programming subproblem:

$$\max_{Y} \quad \sum_{a} value_{t_a}\left(k_a\left[1-\sum_{p\in P,g\in G,d\in D} Pk_{a,c_p,g,d}\bar{R}^b_{a,p,g,d}\right]\right)Y_a \qquad (s0)$$

$$s.t. \quad \sum_{a|l=l_a \wedge m=m_a} Y_a \le missiles_{l,m} \qquad \forall l\in L, m\in M \qquad (s1)$$

$$\sum_{a|t=t_a} Y_a \le max\_missiles_t \qquad \forall t\in T \qquad (s2)$$

$$0 \le Y_a \le 1 \qquad \forall a\in A \qquad (s3)$$

And, given a sub-problem solution $\bar{Y}^b_a$ (that is always integral), the corresponding master problem is:

$$\min_{X,R,Z} \quad Z \qquad (m0)$$

$$s.t. \quad Z + \sum_{\substack{a\in A,p\in P,\\ g\in G,d\in D}} \left[k_a value_{t_a}\bar{Y}^b_a\right] Pk_{a,c_p,g,d}R_{a,p,g,d} \ge \sum_{a\in A} k_a value_{t_a}\bar{Y}^b_a \qquad b=1,2,\ldots \qquad (m1)$$

$$\sum_{g\in G} X_{p,g} \le 1 \qquad \forall p\in P \qquad (m2)$$

$$\sum_{p\in P} X_{p,g} \le 1 \qquad \forall g\in G \qquad (m3)$$

$$\sum_{p\in P,g\in G,d\in D} R_{a,p,g,d} \le 1 \qquad \forall a\in A \qquad (m4)$$

$$\sum_{a\in A,d\in D} salvo_{a,c_p,d,i}R_{a,p,g,d} - loadout_{c_p,i}X_{p,g} \le 0 \qquad \forall p\in P, g\in G_{c_p}, i\in I \qquad (m5)$$

$$X_{p,g} \in \{0,1\} \qquad \forall p\in P, g\in G_{c_p} \qquad (m6)$$

$$R_{a,p,g,d} \in \{0,1\} \qquad \forall a\in A, p\in P, g\in G, d\in D \qquad (m7)$$

$$Z \ge 0 \qquad (m8)$$

For a given defense plan X^*, R^*, we can recover an optimal (integral) attack plan Y^* by solving one additional sub-problem with R fixed at R^*.

9. Bender's Decomposition of the Two-Sided Integer Linear Program With Multi-Cuts

Replacing Z with Z_a, $a \in A$, we can restate the decomposition master problem:

$$\min_{X,R,Z} \sum_{a \in A} Z_a \qquad (c0)$$

$$s.t. \quad Z_a + \sum_{p \in P, g \in G, d \in D} \left(\left[k_a value_{t_a} \overline{Y}_a^b \right] Pk_{a,c_p,g,d} R_{a,p,g,d} \right) \geq k_a value_{t_a} \overline{Y}_a^b$$

$$a \in A \mid \sum_{b=1,2,\ldots} \overline{Y}_a^b \geq 1 \qquad (c1)$$

$$\sum_{g \in G} X_{p,g} \leq 1 \qquad \forall p \in P \qquad (c2)$$

$$\sum_{p \in P} X_{p,g} \leq 1 \qquad \forall g \in G \qquad (c3)$$

$$\sum_{p \in P, g \in G, d \in D} R_{a,p,g,d} \leq 1 \qquad \forall a \in A \qquad (c4)$$

$$\sum_{a \in A, d \in D} salvo_{a,c_p,d,i} R_{a,p,g,d} - loadout_{c_p,i} X_{p,g} \leq 0 \qquad \forall p \in P, g \in G_{c_p} i \in I \qquad (c5)$$

$$X_{p,g} \in \{0,1\} \qquad \forall p \in P, g \in G_{c_p} \qquad (c6)$$

$$R_{a,p,g,d} \in \{0,1\} \qquad \forall a \in A, p \in P, g \in G, d \in D \qquad (c7)$$

$$Z_a \geq 0 \qquad a \in A \qquad (c8)$$

In this master problem, a cut ($c6$) is generated for each attack option a the first time a sub-problem chooses it. If all $|A|$ of these cut constraints are active, they are evidently equivalent to constraints ($t1$). This strong Benders multi-cut decomposition is strongly reminiscent of the "cover cut" decomposition of Israeli and Wood [2002], where each successive master problem is restricted by a cut requiring that the restricted solution differ in at least one detail from any prior solution. E.g.,

$$\sum_{(a \in A, p \in P, g \in G, d \in D) | R^b_{a,p,g,d} = 0} R_{a,p,g,d} + \sum_{(a \in A, p \in P, g \in G, d \in D) | R^b_{a,p,g,d} = 1} (1 - R_{a,p,g,d}) \geq 1 \quad b = 1, 2, \ldots$$

The distinguishing difference between their decomposition and ours is that with each iteration we gain an objective assessment of solution quality --- a lower bound on the optimal solution.

10. Assessing the Value of Defender Secrecy

We have presented one model that assumes perfect defender secrecy, and another than assumes that both attacker and defender have perfect intelligence about each other. We now propose a model that expresses what happens if the defender can keep the locations and intentions of at least some of his interceptor platforms secret.

To do this, partition the defending platforms:

$$p \in P = \{SEEN, SECRET\},\ SEEN \wedge SECRET = \varnothing$$

Using this shorthand for the defender's decision variables and resources, the transparent model is:

$$\min_{\substack{\alpha,\beta,\gamma \\ \{X,R\}\in SEEN}} \sum_{l\in L, m\in M} missiles_{l,m}\alpha_{l,m} + \sum_{t\in T} max\_missiles_t \beta_t + \sum_{a\in A}\gamma_a \qquad (v0)$$

$$s.t. \quad \alpha_{l_a,m_a} + \beta_{t_a} + \gamma_a + \sum_{p\in SEEN, g\in G, d\in D} k_a value_{t_a} Pk_{a,c_p,g,d} R_{a,p,g,d} \ge k_a value_{t_a} \qquad \forall a \in A \qquad (v1)$$

$$\sum_{g\in G} X_{p,g} \le 1 \qquad \forall p \in SEEN \qquad (v2)$$

$$\sum_{p\in SEEN} X_{p,g} \le 1 \qquad \forall g \in G \qquad (v3)$$

$$\sum_{p\in SEEN, g\in G, d\in D} R_{a,p,g,d} \le 1 \qquad \forall a \in A \qquad (v4)$$

$$\sum_{a\in A, d\in D} salvo_{a,c_p,d,i} R_{a,p,g,d} - loadout_{c_p,i} X_{p,g} \le 0 \qquad \forall p \in SEEN, g \in G_{c_p}, i \in I \qquad (v5)$$

$$\alpha_{l,m} \ge 0 \qquad \forall l \in L, m \in M \qquad (v6)$$

$$\beta_t \ge 0 \qquad \forall t \in T \qquad (v7)$$

$$\gamma_a \ge 0 \qquad \forall a \in A \qquad (v8)$$

$$X_{p,g} \in \{0,1\} \qquad \forall p \in SEEN, g \in G_{c_p} \qquad (v9)$$

$$R_{a,p,g,d} \in \{0,1\} \qquad \forall a \in A, p \in SEEN, g \in G, d \in D \qquad (v10)$$

Given two-sided transparent interceptor commitments R^*, induce the associated attack plan by solving, e.g., ($s0$ – $s3$) with $\overline{R} = R^*$ for Y^*, and locate secret platforms and intercept leakers by:

$$\min_{\{X,R\}\in SECRET} \sum_{\substack{a\in A \\ \left| Y_a^*=1 \wedge \sum_{p\in SEEN, g\in G, d\in D} R_{a,p,g,d}=0 \right.}} value_{t_a}\left(k_a\left[1-\sum_{p\in SECRET, g\in G, d\in D} Pk_{a,c_p,g,d} R_{a,p,g,d}\right]\right); (h0)$$

$$\sum_{g\in G} X_{p,g} \le 1 \qquad \forall p\in SECRET \qquad (h1)$$

$$\sum_{p\in SECRET} X_{p,g} \le 1-\sum_{p\in SEEN} X^*_{p,g} \qquad \forall g\in G \qquad (h2)$$

$$\sum_{p\in SECRET, g\in G, d\in D} R_{a,p,g,d} \le 1 \qquad \forall a\in A \qquad (h3)$$

$$\sum_{a\in A, d\in D} salvo_{a,c_p,d,i} R_{a,p,g,d} \le loadout_{c_p,i} X_{p,g} \qquad \forall p\in SECRET, g\in G_{c_p}, i\in I \qquad (h4)$$

$$X_{p,g}\in\{0,1\} \qquad \forall p\in SECRET, g\in G \qquad (h5)$$

$$R_{a,p,g,d}\in\{0,1\} \qquad \forall a\in A, p\in SECRET, g\in G, d\in D \qquad (h6)$$

The resulting mathematical formulation is an integer linear program that recommends optimal stationing locations and interdictions for defender assets by minimizing the enemies' ability to inflict damage. Defender optimal interdiction strategy accounts for the launch sites of the attack, the missile types used and the targets attacked. Additionally, we balance interceptor capabilities and defender platform inventory to minimize the expected number of enemy ballistic missiles that penetrate the air defenses. Defender interdiction strategy is further constrained by linking interceptor capabilities to the oblate spherical triangle formed by the geographic coordinates of the attacker launch site, target, and defender location, which depend on the attack the enemy chooses in the assignment model. The result is an integer linear program that recommends optimal stationing positions for defender platforms that minimize the maximum expected damage of an enemy attack. The goal is to produce a provably optimal, face-valid defender plan on a portable computer.

C. A NOTIONAL NORTH KOREAN SCENARIO, CIRCA 2010

There is no perfect defense system, so we anticipate that some fraction of the attacker's ballistic missiles will penetrate our air defense. Friendly forces will intercept

incoming ballistic missiles until they are all destroyed or until the intercepting units exhaust their capacity and are overwhelmed by the incoming strike. Defending forces will attempt to minimize the number of ballistic missiles that are not intercepted and reach their intended target; thereby minimizing the expected damage inflicted. *The goal of the joint forces commander is to station defending interceptors in positions that minimize, in some sense, damage to the defended targets.*

We have developed a North Korean scenario, circa 2010, in which we specify a North Korean arsenal of ballistic missiles and launch sites, a U.S. contingent of ballistic missile defense platforms, and a list of targets with associated target values. We use this scenario to evaluate proposed theater ballistic missile defense options. In developing this scenario we have made some assumptions. The following data are used for our planning scenario and remain fixed throughout the analysis.

1. Attacker Launch Sites

The attacker candidate launch sites are based upon actual North Korean known missile facilities and bases from unclassified sources [FAS 2003b]. Table 2 enumerates the notional North Korean missile launch sites, and Figure 6 shows the approximate locations overlaid on a map of North Korea.

Launch Sites	Latitude (N)		Longitude (E)	
	Degrees	Minutes	Degrees	Minutes
Chiha-ri	38	37	126	41
Chunggang-up	41	46	126	53
Kanggamchan	40	24	125	12
Kanggye	40	7	126	35
Man'gyongdae-ri	38	59	125	40
Mayang	40	0	128	11
Namgung-ni	39	8	125	46
No-dong	40	50	129	40
Ok'pyong	39	17	127	18
Paegun	39	58	124	35
Pyongyang	39	0	125	45
Sangwon	38	50	126	5
Sunchon	39	25	125	55
Tokch'on	39	45	126	15
Toksong	40	25	128	10
Yongo-dong	41	59	129	58

Table 1. North Korean scenario launch sites (after fas.org[2003b])
These launch sites are current missile production facilities and missile bases, and are used in our scenario as potential launch sites.

Figure 6. North Korean launch locations

Approximate positions of the North Korean launch sites convey the geographical complexity of the scenario.

We distill unclassified sources and notionally position numbers of each type of North Korean missile at each candidate launch site. We assume that the enemy will fire any number and types of missiles that will maximize damage.

2. Attacker Missiles

The following missiles are selected from the North Korean inventory. Missile characteristics are notional, compiled from unclassified sources [e.g., Jane's 2003d]. Each missile is assumed to be 100-percent reliable, and if not intercepted has a 100-percent probability of hit (k_a=1.0) against its intended target. This expresses the worst-case situation. Table 3 displays the minimum and maximum ranges (in kilometers) of our scenario ballistic missiles.

Missile	Range (km) Minimum	Maximum
Scud-B	40	330
Scud-C	40	700
No-Dong	1,350	1,500
Taep'o-Dong I	2,200	2,900
Taep'o-Dong II	3,500	4,300

Table 2. North Korean ballistic missile types with their range limits

a. Missiles Available

Table 3 lists the maximum number and type of North Korean ballistic missiles allocated to each launch location.

Launch Sites	Scud-B	Scud-C	No-Dong	Taepo-Dong I	Taepo-Dong II
Chiha-ri	15	20	10		
Chunggang-up		10	10		
Kanggamchan		15	10		
Kanggye		15	10		
Man'gyongdae-ri	10	20	10		
Mayang		15	20	1	1
Namgung-ni	5	15	2		
No-dong		5	15	1	1
Ok'pyong	15	15	10		
Paegun		15	10		
Pyongyang	15	15	10		
Sangwon	15	20	10		
Sunchon	5	15	10		
Tokch'on	5	15	15		
Toksong	5	15	15		
Yongo-dong			20	1	1

Table 3. North Korean ballistic missiles available by launch location.

b. Maximum Attacks

We assess an attack that allows a defended asset *target* to be attacked at most once. Increasing the number of attacks per target is trivial.

3. Targets on a Defended Asset List

Figure 7 displays the defended asset list (DAL) and target values for our scenario. Estimating target value is important because we need to infer the attacker's motives. We assume that the attacker will have some foreknowledge of our strengths, weaknesses and critical nodes and will attack targets that are important to us. Representing target values that have meaning is an important preparatory step to get sensible defense results.

We generate target values based upon a subjective assessment of the four factors currently used in air defense planning: criticality, vulnerability, reconstitutability and threat [Army 2004].

a. Criticality (c)

Criticality is a judgment of the degree to which one of our defended assets is essential to us. A high value indicates that the asset is extremely critical to us. A low value indicates otherwise.

b. Vulnerability (v)

Vulnerability is an evaluation of the degree to which a target is susceptible to an air or missile attack or is vulnerable to surveillance. A high value indicates that the target is extremely vulnerable, unprotected and in the open with clear lines of approach. A low value indicates otherwise.

c. Reconstitutability (r)

Reconstitutability is an assessment of the degree to which the target can recover from inflicted damage in terms of time, equipment, and available manpower to resume normal operation. A high value indicates that the target would need considerable time, equipment and/or manpower to return to normal operation following an attack. A low value indicates otherwise.

d. Threat (t)

Threat is an estimate of the probability of our asset being attacked by our enemy. A high value indicates that it is nearly certain that the enemy is or will attack this target. A low value indicates otherwise.

Combining these factors, our target $value_t$, is:

$$value_t = \ln\left(c * v * r * t\right) + 1,$$

where c, v, r, t range from 1-10 and $value_t$ can range from 1.0 to slightly more than 10.0. The natural log function (ln) was chosen to convert the product of c, v, r, and t units to a target utility that retains partial order between any target pair and exhibits a range of values not too unlike those of each factor.

Target	Criticality	Vulnerability	Reconstitutability	Threat	Target Value
Seoul	4	8	5	9	8.3
Pusan	8	7	8	10	9.4
Inchon	3	6	5	4	6.9
Chinhae	7	7	7	8	8.9
OsanAB	10	8	9	10	9.9
Kunsan	10	7	9	10	9.7
Tokyo	4	9	4	7	7.9
Yokosuka	8	8	7	7	9.1
Sasebo	7	8	7	7	8.9
Okinawa	7	7	8	3	8.1
Misawa	8	5	7	5	8.2
Atsugi	4	7	6	5	7.7

Figure 7. Targets on a Defended Asset List (DAL)

Targets are on this list because of their obvious political or military significance and are spread out over South Korea, Japan and Okinawa. Each target is assigned four scores, respectively reflecting criticality, vulnerability, reconstitutability and threat. For example, Seoul has (c,v,r,t) values of (4,8,5,9), which result in a target value of $\ln(4*8*5*9)+1=8.3$.

4. Ballistic Missile Accuracy and Target Value Relationship

The United States categorizes ballistic missiles into four categories based upon missile maximum ranges: short-range (less than 1000 km), medium-range (less than 3000 km), intermediate-range (less than 5500 km) and intercontinental (greater than 5500 km) [FAS 2003a].

Another characteristic of every missile is an evaluation of its accuracy, usually expressed as its circular error probable (CEP), or the radial distance within which a missile will impact fifty percent of the time.

For precision-guided munitions the CEP should be very small, say within meters. A weapon with a large CEP, say hundreds of meters, has a much smaller chance of hitting its intended target – which equates to a greater probability of generating unwanted collateral damage.

For illustrative purposes, we compare a notional missile at two different CEP levels, 30m and 1000m, and calculate the probability that the missile will land within its lethal radius of a fixed target.

Lethal radius is generally determined by the amount of peak overpressure generated by the explosion in pounds per square inch (psi). (This odd juxtaposition of English with metric measure is ubiquitous in the weapons effects literature.) From "Introduction to Naval Weapons Engineering" [FAS 2004], we find that a level of 3 psi peak overpressure is generally considered to be enough to cause moderate damage to troops in the open and to parked aircraft. The distance at which 1kg of TNT will produce a 3psi peak overpressure is 6 meters. Consider a notional missile with a 1000 kg Composition-B warhead. From the Berthelot approximation, a 1000kg Comp-B warhead is equivalent to a 1495kg TNT warhead. To find the lethal radius of our 1495kg TNT-equivalent warhead we use the *scaling law* that is defined as

$d_W = d_0 W^{\frac{1}{3}}$ where d_0 is the distance from 1 kg TNT and d_W is the equivalent distance from the W kg of TNT explosive.

$$
\begin{aligned}
\textit{Lethal Radius} &= (\text{1 kg TNT distance (meters)}) * \sqrt[3]{\text{TNT equivelant warhead weight(kg)}} \\
&= 6 * \sqrt[3]{1495} \approx 69 \text{ meters.}
\end{aligned}
$$

We assert that for theater ballistic missiles, accuracy may not be that important. Even if the missile does not impact its exact target, if it is not intercepted it will impact somewhere. Figures 8 and 9 illustrate a 100-shot scatter plot of impact points around a target superimposed on an aerial photograph of the former Clark Air Base in the Philippines.

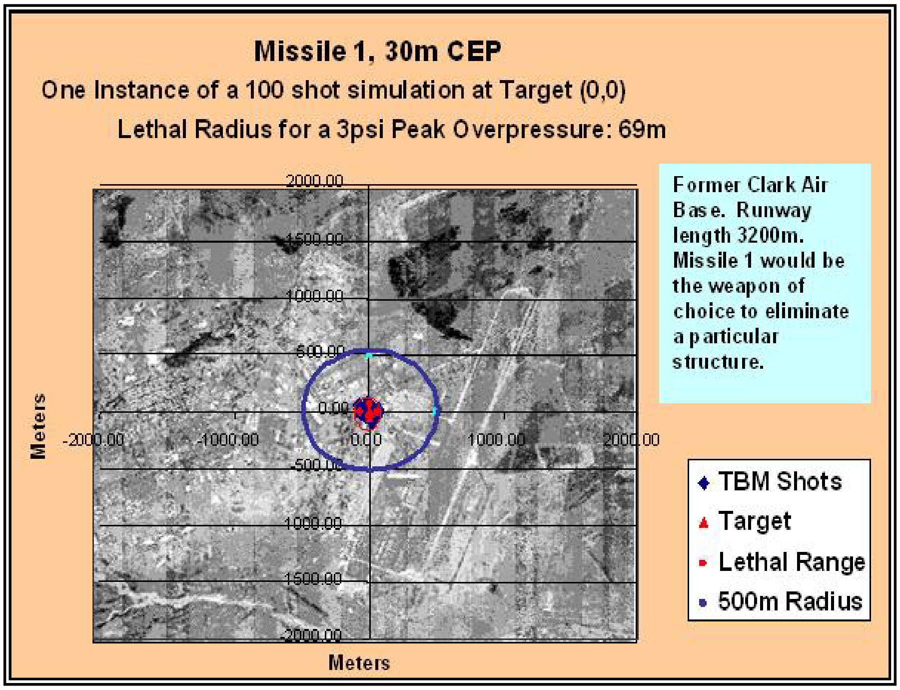

Figure 8. 30-meter circular error probable missile impact points

The inner circle represents a 69m lethal radius; the outer ring is a 500m ring. Nearly all 100 shots landed within the lethal radius.

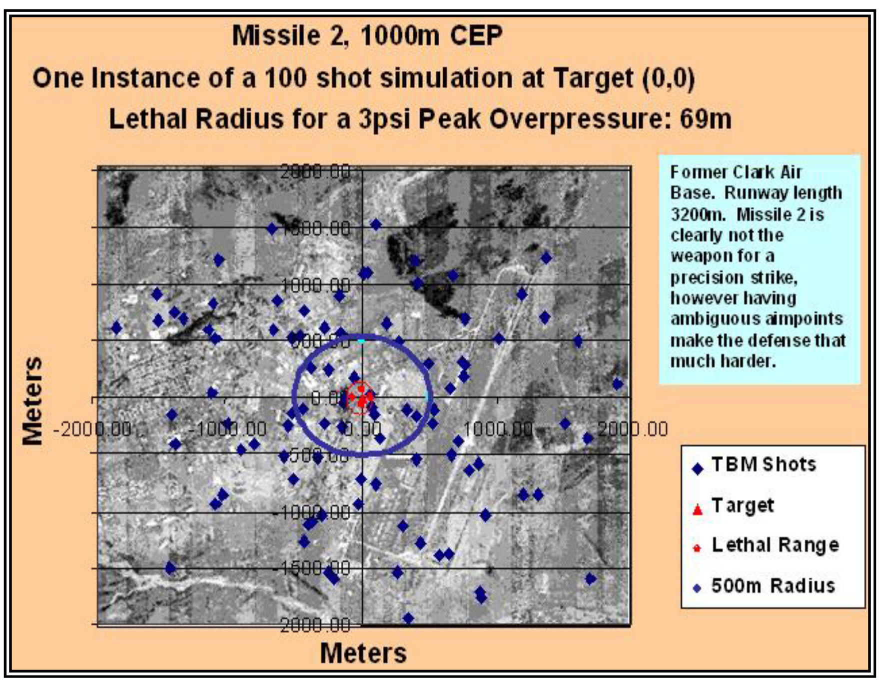

Figure 9. 1000-meter circular error probable missile impact points

The inner circle represents a 69m lethal radius, the outer circle is a 500m ring. Notice the majority of missile impact points are outside the 500m circle.

Certainly we can expect that lower CEP-weapons will be more effective against specific military targets, but our enemies will use ballistic missiles as weapons of terror – weapons designed to inflict a large amount of damage and destruction indiscriminately. If a ballistic missile is armed with a nuclear, biological or chemical warhead, it does not matter much if it hits an exact aim point: close is good enough.

5. Defense Platforms

For our 2010 defense design we have two AEGIS cruisers each, with 10 SM3 and 20 SM2 interceptors, and one AEGIS destroyer with 20 SM2 interceptors. We assume that each AEGIS ship is configured for ballistic missile defense and deployed as an independent entity. Additionally, we assume that each AEGIS platform of each ship class is allocated the same interceptor loadout within its same class, i.e. cruiser or destroyer. Varying the interceptor loadout by platform is trivial.

We can use one PATRIOT battery. We assume that a deployed PATRIOT battery consists of 8 mobile launchers and associated support vehicles, and that each mobile launcher is loaded with four PAC-3 missiles, two PAC-2 GEM missiles, and one PAC-2 missile. Varying the mix of missiles by battery, or the composition of the battery itself, is trivial.

There is also one THAAD battery. We assume that a deployed THAAD battery consists of a mobile launcher containing 10 interceptors and the associated radar and support vehicles. Varying the number of interceptors by battery is trivial.

6. Interceptor Ranges

Table 4 specifies the maximum range of the various interceptors used by our defense platforms in our scenario. Ranges were gleaned from open literature [Jane's 2003b,c,e].

Interceptor	Maximum Range (km)
Thaad	250
PAC-2	160
PAC-2GEM	160
PAC-3	70
SM2 blk III variants	120
SM3	1,200

Table 4. Defender interceptor missile ranges

7. Interceptor Effectiveness: Probability of Kill (Pk)

An attacking ballistic missile may follow a "flyout" trajectory that prevents certain interceptors from being able to engage from certain defense points. For example, a ballistic missile having a range less than 1000 km does not reach sufficient altitude for midcourse interceptors to be effective; therefore such a missile must be engaged by terminal defense systems. Navy AEGIS ships with a high altitude-extended range missile and Army THAAD batteries are being developed as our midcourse defense systems. Our terminal defense systems are the Army PATRIOT and AEGIS ships using medium range missile variants.

Our probability of Kill (Pk) is zero or a constant value if feasible engagement conditions are met. We can also represent interceptor effectiveness with a Pk table indexed by cross-range and down-range proximity, and by "flyout" altitude. Pk tables condense into a single number the complex relationship between attack launch site, target of attack, attacker missile type, defender position in relation to the attacking missile trajectory, and the interceptor(s) available to the defender to use to thwart the attack.

8. Candidate Interceptor Locations

Our scenario is discretized into a latitude and longitude grid to the nearest degree (see Figure 10). This gives us an approximate sixty nautical mile fidelity with four hundred twenty (420) candidate locations; each platform may be positioned at any of an appropriate subset of these candidate locations.

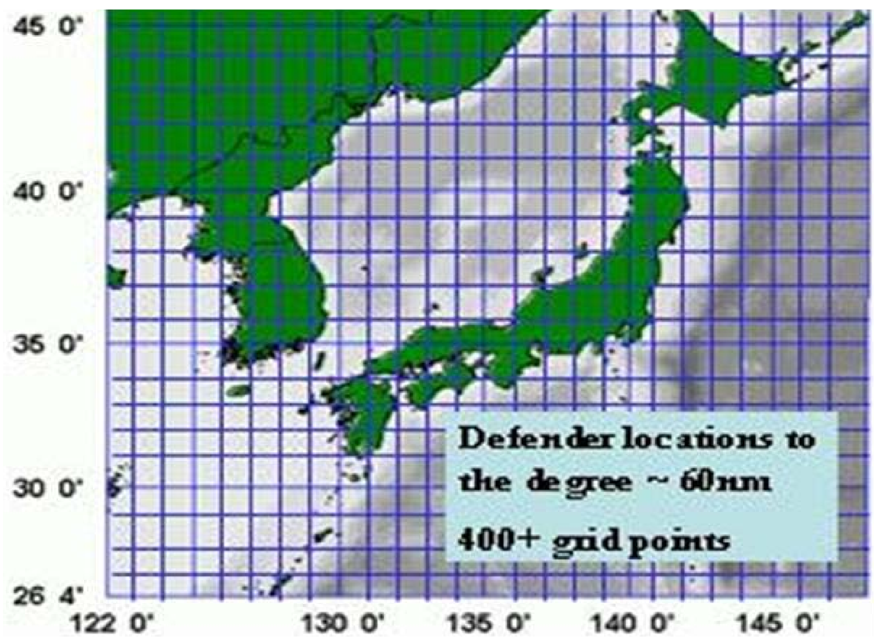

Figure 10. Candidate defender interceptor locations

Our grid points are placed at each integer latitude and longitude, and each candidate defending platform may be located at a subset of these points (e.g., ships only at sea and land-based units on land).

D. MEASURE OF EFFECTIVENESS

In our scenario we assess the maximum damage an enemy attack can inflict on undefended targets given attacker resource constraints. We then evaluate an optimal interception plan for this enemy maximal attack subject to defender resource constraints *assuming the enemy does not know we are making such plans*. The difference between the expected damage from the undefended attack and the expected damage of the same attack against unobserved defenders is the *value of defender secrecy*.

Next, we assume that the enemy can see our defensive preparations and that we can see his attack preparations, and evaluate the mutually optimal plans that this transparency suggests to the attacker and defender. The resultant expected damage is the *value of transparency*, or more practically – the value of assuming our enemy is smart and will attack in a manner that exploits our weaknesses.

Finally, we evaluate cases where we may be able to keep some defending platforms hidden from the enemy, while others can be seen by the attacker. This adds nuance to the value of defender secrecy.

III. RESULTS AND ANALYSIS

The ability to express this problem as an integer linear program enables us to objectively assess solution quality. I.e., given our assumptions and data, we can establish with absolute certainty how much better any defensive plan might be that we have not already discovered.

A. A MAXIMAL UNDEFENDED ENEMY ATTACK

Table 5 represents an optimal attack that fires a single missile at each undefended target producing a total expected damage of 93.1.

Launch Site	Missile Type	Target
Chihari	ScudC	Sasebo
Chihari	NoDong1	Okinawa
Kanggamchan	ScudC	Seoul
Kanggamchan	ScudC	Chinhae
Kanggamchan	NoDong1	Tokyo
Kanggamchan	NoDong1	Yokosuka
Kanggamchan	NoDong1	Atsugi
Okpyong	ScudC	Kunsan
Paegun	NoDong1	Misawa
Pyongyang	ScudC	Pusan
Tokchon	ScudC	Inchon

Table 5. Optimal North Korean attack

There are no interceptions at all. Each target is attacked with a single missile producing an expected damage of 93.1.

Figure 11 illustrates what this attack looks like when the launch sites are connected to the targets as depicted on a map of the theater.

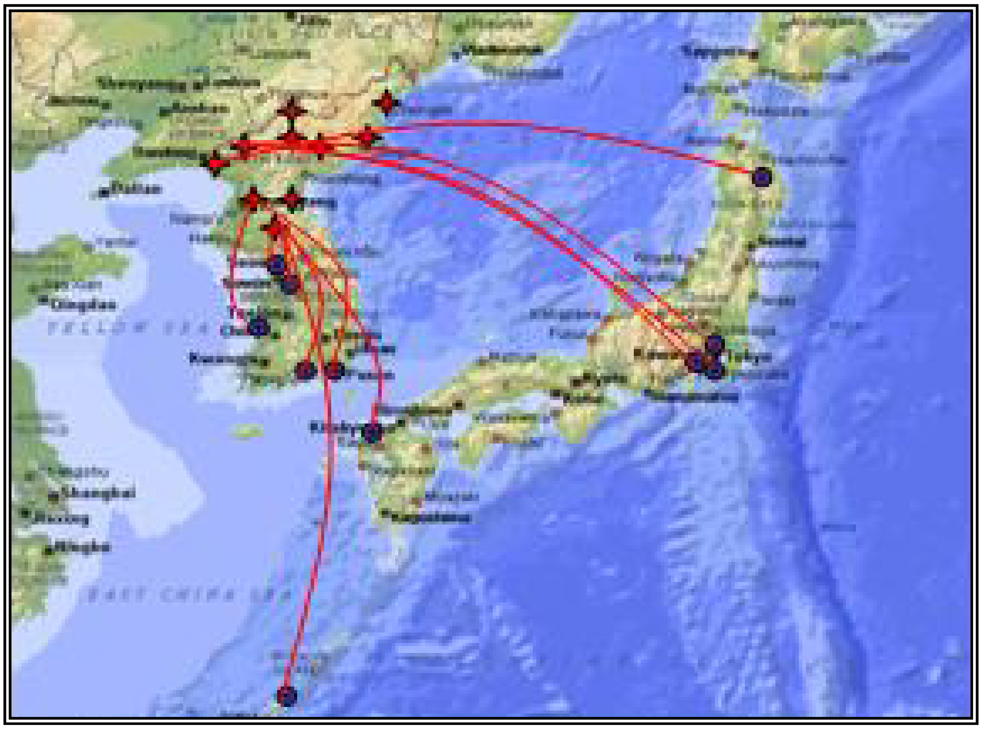

Figure 11. Theater-wide optimal attack

A maximal attack is shown with at most one missile aimed at each target and no interceptions. Maximum expected damage is 93.1.

B. AN OPTIMAL DEFENSE DESIGN

Assuming that the attacker does not observe our defensive preparations, we position our assets to intercept his optimal attack and reduce his attack to an expected damage value of 0.93. Our defenders know about all attacks ahead of time, so they are able to position themselves and engage attacker missiles with interceptors that have a high probability of kill. A reduction in damage to 0.93 equates to approximately one-tenth of a missile leaking through.

Defender Class	Platform	Latitude	Logitude
AegisCG	CG01	35	125
AegisCG	CG02	31	128
AegisDDG	DDG01	34	126
Patriot	Pbat1	35	129
Thaad	Tbat1	36	136

Table 6. Optimal defender locations maintaining defense secrecy

From these (hidden) positions, defending platforms intercept every incoming missile, but do not necessarily kill it. The maximum expected damage is reduced to 0.93, or about one-tenth of a missile leaking through.

Figure 12 illustrates the defender positions relative to the attack.

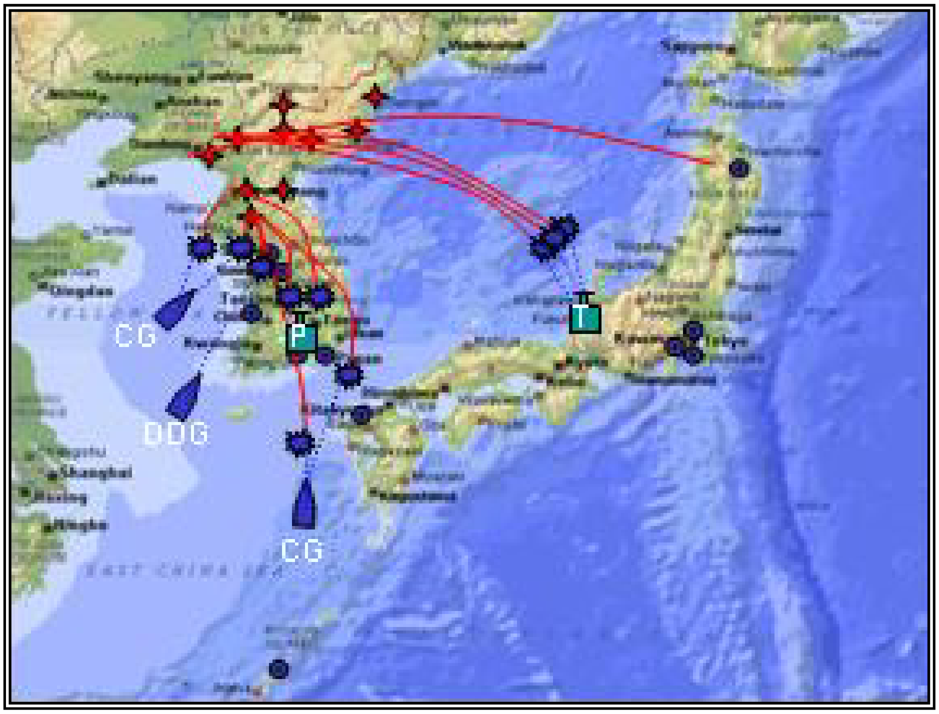

Figure 12. Optimal defense, attacker unaware
This illustrates interceptor engagements by hidden defenders. Expected damage is reduced to 0.93.

C. ASSUME TRANSPARENCY: A TWO-SIDED OPTIMIZATION

If each side can observe what the other is doing, we find ourselves in a position where the attacker knows we may commit an interceptor *salvo* to each candidate missile attack, and shoot it if he launches that attack. The defender knows that the attacker will get some of his missiles through. The objective for the defense is to minimize the maximum expected damage, given the attacker can see and take advantage of our pre-positioned forces.

The two-sided attack and defense produces a maximal attack with expected damage of 6.1, *an overall reduction in expected damage of 93.4 percent*. Table 7 and 8 illustrate the attack and defense, respectively, and Figure 13 shows what the attack and defense look like on a map of the theater.

Launch Location	Missile Type	Target
Kanggamchan	ScudC	Seoul
Pyongyang	ScudC	Pusan
Tokchon	ScudC	Inchon
Kanggamchan	ScudC	Chinhae
Okpyong	ScudC	Kunsan
Kanggamchan	NoDong1	Tokyo
Kanggamchan	NoDong1	Yokosuka
Kanggamchan	NoDong1	Misawa
Kanggamchan	NoDong1	Atsugi
Paegun	NoDong1	Misawa
Chihari	NoDong1	Okinawa

Table 7. Optimal attack given transparency between defender and attacker
Each target is attacked with at most one missile. The attacker knows the defender locations and whether the defender has committed interceptor resources to thwart the attack.

Defender Class	Platform	Latitude	Longitude
AegisCG	CG47	39	130
AegisCG	CG48	34	129
AegisDDG	DDG68	38	130
Patriot	Pbat1	36	128
Thaad	Tbat1	37	138

Table 8. Optimal defense given transparency between attacker and defender
Each defender platform is located to minimize the attacker's worst possible attack. The defender has committed missiles to thwart potential attacks that may not actually be launched, but will be intercepted if they are.

Figure 13. Optimal two-sided attack and defense design

Given attacker-defender transparency, expected damage is 6.1, an overall reduction of 93.4 percent. Defenders have committed interceptors to potential attacks that are not launched for precisely this reason.

D. ASSUME PARTIAL TRANSPARENCY

Suppose that we can keep our naval defender platforms hidden from the attacker, knowing that he can observe our land-based defenders. The resulting expected damage moves from the upper bound of total transparency towards the lower bound of total defender secrecy. The difference between the expected damage in the transparent solution and the expected damage of this solution is the value of partial defender secrecy. In practical terms this value quantifies how an increase in information hiding effort, either through funding, tactics or a combination of both, will reduce the attacker's ability to inflict damage.

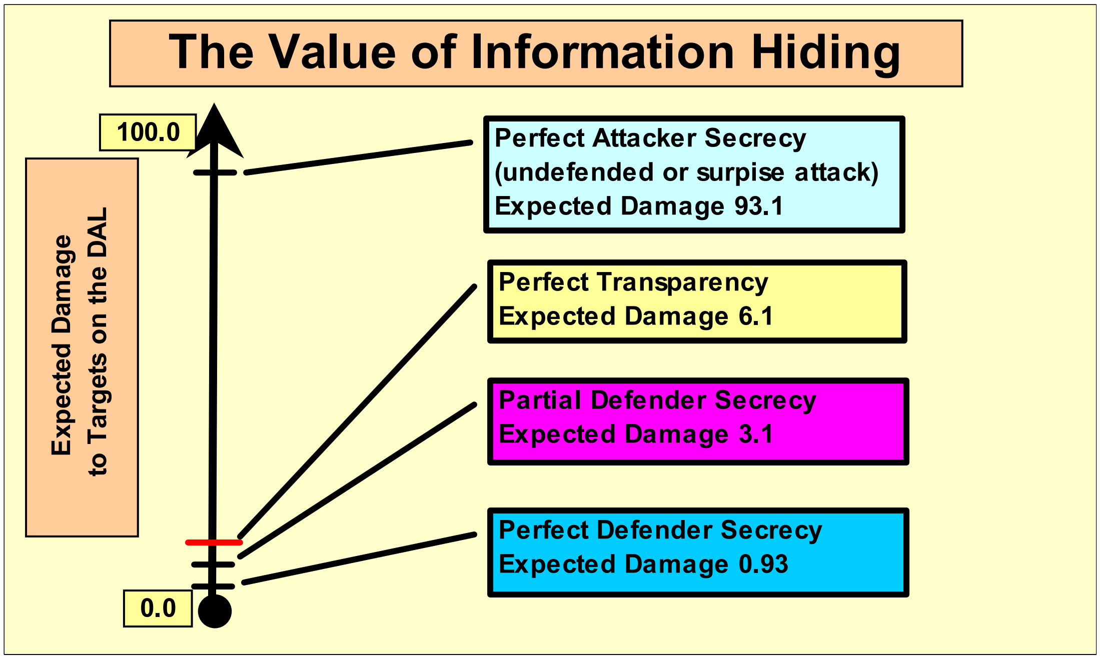

Figure 14. The value of attacker secrecy, and defender secrecy

This illustrates the expected damage under different amounts of attacker and defender secrecy. The value of secrecy is the positive difference between the expected damage under that level of secrecy and the expected damage in the transparent model (e.g., the value of perfect defender secrecy is 6.1 - .93 = 5.17).

The value of partial defender secrecy is the difference between the expected damage of the perfect transparent solution and the expected damage of the perfect defender secrecy solution where we know what the attacks will be and keep all of our defender's hidden from the attacker (e.g. $6.1 - 0.93 = 5.17$).

The more effort the defender expends to keep his platforms hidden, the closer he gets to the lower bound of perfect defender secrecy.

IV. CONCLUSIONS AND RECOMMENDATIONS

Although providing a quick decision aid in the past, the visual "launch fan" and "interceptor envelope" cookie-cutter model is too restrictive for our use. We see little to recommend using such a device when an exigent scenario may involve an arbitrary field of launch sites, a variety of attack missile types, and a dispersed defended asset list of potential targets. A launch fan expresses the feasible range of attack tracks of one missile type from one launch site to any vulnerable target location. For our purposes, we account for every candidate "attack trajectory" from launch site to target location. It is important to change the paradigms of "launch fans" and "cookie-cutter" interceptor coverage zones to reflect an enemy that is smart and capable of knowing and exploiting our weaknesses and defense strategies. The controversy here centers on whether we score our interceptions by just killing attacking missiles, or by defending vulnerable targets on our defended asset list. We choose the latter.

We have produced an integer linear program that expresses this problem using well-established methods to selectively enumerate and qualitatively assess solution quality. We model missile attacks with an assignment that maximizes the expected damage of an attack despite some defense interception plan. We assume the enemy knows our defended asset list (DAL), agrees with these targets, and can see what our defensive prepositions will be.

We propose a decision support tool that can offer provably optimal interception plans on a laptop computer in minutes. These integer linear programs can be solved faster, and can be expected to find a near-optimal solution. The space and power requirements for our solution are trivial. The Joint Task Force's Area Air Defense Commander (AADC) and Regional Air Defense Commander (RADC) may use this decision support tool for initial defense planning and assessment. Our model gives the commander the ability to qualitatively assess the value of hiding information from the attacker. In addition it could provide insight to the ballistic missile defense (BMD) program officers in Washington.

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LIST OF REFERENCES

Central Intelligence Agency [2001] Foreign Missile Developments and the Ballistic Missile Threat Through 2015. Unclassified summary of the National Intelligence Estimate produced by the National Intelligence Council.

Department of the Army Field Manual FM 3-01.11 [2003] Appendix A – ADA Employment Principles, Guidelines, and Priorities, accessed 2 December at http://www.globalsecurity.org/military/library/policy/army/fm/3-01-11/appa.html.

Department of Defense [2004], Fiscal 2005 Department of Defense Budget Release, 2 February, accessed 19 March 2004 at http://east.defenselink.mil/releases/2004/nr20040202-0301.html

Federation of American Scientists [2003a] "Ballistic Missile Basics," accessed 21 May at http://www.fas.org/nuke/intro/missile/basics.html.

Federation of American Scientists [2003b] DPRK - Special Weapons Facilities, accessed 13 April at http://www.fas.org/nuke/guide/dprk/facility/index.html.

Federation of American Scientists [2004] "Introduction to Naval Weapons Engineering," accessed 15 January at http://www.fas.org/man/dod-101/navy/docs/es310/chemstry/chemstry.html.

Israeli, E. and Wood, R.K. [2002], "Shortest-Path Network Interdiction," *Networks*, Vol. 40, pp. 97-111.

Jane's C4I Systems, Command Information Systems - Naval, United States [2003a], accessed 19 May at www4.janes.com.

Jane's Electro-Optic Systems 2002-2003, Naval Systems – Surface-to-Air Missiles, United States [2003b] accessed 19 May at www4.janes.com.

Jane's Land Base Air Defense, Static and Towed Surface-to-Air Missile Systems, United States [2003c] accessed 19 May at www4.janes.com.

Jane's Strategic Weapon Systems 38, Offensive Weapons, Korea, North [2003d] accessed 19 May at www4.janes.com.

Jane's Strategic Weapon Systems 39, Defensive Weapons, United States [2003e] accessed 19 May at www4.janes.com.

SPARTA, Inc [2004] *Commander's Analysis and Planning Simulation* (CAPS) v 8.0 Users Manual.

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